

CMB dipole revisited

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Abstract

We re-evaluate the build-up of a horizon-sized temperature profile of amplitude $\delta T/T \sim 10^{-3}$ at $z \sim 1$ in light of an improved determination of the black-body anomaly, based on a pure SU(2) Yang-Mills theory, and the correction of a mistake in deriving the evolution equation for δT . Our present results for the temperature profiles hardly are distinguishable from those published previously.

1 Introduction

Judged by Standard-Model physics (photon propagation based on a U(1) gauge group, cosmologically scale invariant primordial perturbation spectrum) the observation that the dipole-subtracted CMB TT correlation function $C(\theta)$ is consistent with zero for angular separation of 60 degrees and larger and that the low-lying CMB multipoles ($l = 2, 3$) seem to be statistically aligned are incomprehensible [1]. This appears to be in line with recent radio-source investigations [2] in the neighbourhood of the CMB cold spot (on 4 and 10 degrees resolution one observes an amplitude of $-73 \mu\text{K}$ and $-20 \mu\text{K}$, respectively) which suggest that this cold spot is not of any primordial origin. Rather, it seems to be related to physics operating at a redshift of about $z \sim 1$. When invoking the late integrated Sachs-Wolfe effect this physics seems to have created a region void of matter of $\sim 140 \text{ Mpc}$ radius. Notice, however, that there may be an alternative explanation, which does not relate to the spatial distribution of gravitational potentials but to the occurrence of a low-temperature, low-frequency anomaly in black-body spectra generated by the nonabelian effects of an SU(2) gauge symmetry underlying the *propagation* of thermalized photons [3]. It is interesting though that the late integrated Sachs-Wolfe effect points to an anomaly which is operative at the same value of redshift $z \sim 1$ where the black-body anomaly is maximal.

The approach of [3] is to treat the temperature fluctuation δT as a scalar field whose evolution is subject to primordial, Gaussian initial conditions at sufficiently high redshift z . Doing justice to luminosity-redshift curves extracted from calibrated supernovae and to best-fitted large- l CMB data, a standard ΛCDM background cosmology was used in [3] to drive the Universe's expansion. In this given cosmological background, the evolution of δT is, according to [3], in addition determined by coefficient functions provided by the modified black-body spectrum. These functions enter into an evolution equation derivable from an action principle with an assumption about the relative normalization of kinetic versus potential term in the action being made. So far, this normalization is determined empirically although it should, in principle, be derivable from the SU(2) gauge theory of scale $\Lambda_{\text{CMB}} \sim 10^{-4} \text{ eV}$ postulated to underly photon propagation [4, 5, 6, 7, 8, 9].

As a consequence, the rapid build-up of a horizon-sized δT profile of CMB dipole strength is predicted to occur at $z \sim 1$. In general, the dipole subtracted angular TT correlation function $C(\theta)$ is obtained by integrating the map of fluctuations δT at $z = 0$ along two lines of sight which are separated by the angle θ and by subsequently averaging over maps subject to varying initial conditions and over pairs of lines of sight¹.

The purpose of the present work is to investigate to what extent the results in [3] are modified when subjecting the z evolution of the δT maps to the exact black-body anomaly computed in [9] and to a corrected evolution equation for δT . Notice that in [3] a black-body anomaly was computed based on the approximation that

¹These maps are dipole subtracted.

the photon is on its naive mass shell ($p^2 = 0$) [6]. Compared with the selfconsistent determination of the screening function G [9] this yields good results for $T \leq 2T_c$ where T_c denotes the critical temperature for the deconfining-preconfining phase transition. However, for $T > 2T_c$ the exact (selfconsistently determined) result for G falls off much faster with increasing T and the spectral gap closes more rapidly than the results obtained with $p^2 = 0$ seemed to suggest.

The paper is organized as follows. In Sec. 2 we briefly discuss the objects that are essential to our present investigation, namely, the accurate determination of the spectral black-body anomaly and the cosmological evolution equation for the temperature fluctuation δT . In Sec. 3 we compare the old results, obtained with the erroneous evolution equation and an inaccurate black-body anomaly, with the ones obtained by rectifying these problems observing that on the level of spatial spherical symmetry practically no difference occurs. Finally, we give a short summary in Sec. 4.

2 Black-body anomaly and cosmological evolution of a spherical temperature profile

In [6] the polarization tensor of the massless mode was computed to one-loop accuracy in deconfining SU(2) Yang-Mills thermodynamics under the assumption that the external four-momentum p is on the free mass shell: $p^2 = 0$. To address the effects determining the photon dispersion law fully² the modified on-shell dispersion relation, as it arises from the resummed one-loop polarization, needs to be taken into account in a selfconsistent way. This was done in [9].

More precisely, one has

$$\omega^2 = \mathbf{p}^2 + G(|\mathbf{p}|, T), \quad (1)$$

where ω denotes the frequency, \mathbf{p} spatial momentum, and the function G can be positive (screening) or negative (screening). It was shown in [9] and also in [10] that within any experimentally feasible accuracy the selfconsistent determination of G only involves the one-loop tadpole diagram³.

The implications of this result for the low-frequency ($\omega \leq 0.15 T$) and low temperature ($T_c \leq T \leq 5 T_c$) regime of the associated black-body spectrum were discussed in [7] for $p^2 = 0$ and in [9] for $p^2 = G$ after postulating that an SU(2) gauge

²It was shown in [10] that three-loop contributions to the pressure of deconfining SU(2) Yang-Mills thermodynamics are suppressed as compared to the two-loop correction by a factor of 10^{-3} or smaller. This result relates to the photon polarization by cutting the massless lines in the corresponding pressure corrections. Thus practically no modification of the one-loop result for the photon polarization arises from higher loops.

³A potential imaginary contribution to G , arising from the other one-loop diagram, vanishes identically on the mass shell in Eq. (1) [9].

principle underlies photon *propagation*. For $T \leq 2T_c$ the former and the later results coincide while at higher temperatures the selfconsistent calculation indicates a decay $\propto T^{-1/2}$ of the spectral gap as compared to the result $\propto T^{1/3}$ obtained by assuming $p^2 = 0$ [6].

To avoid a contradiction with precision measurements of the present CMB intensity observing a perfect U(1) (Planckian) spectrum (spectral deviations leading to $\delta T/T \sim 10^{-5}$ only), the Yang-Mills scale Λ_{CMB} of this SU(2) theory must be such that the present CMB temperature $T_{\text{CMB}} \sim 2.73 \text{ K}$ coincides with the critical temperature T_c of the deconfining-preconfining phase transition [13].

In the SU(2) based black-body spectrum, which exhibits its largest deviation from the Planckian spectrum at $T \sim 2T_c$, there is an exponentially with increasing frequency decaying regime of antiscreening (photon's energy less than the modulus of its spatial momentum). At low frequencies the modified black-body spectrum exhibits screening (photon's energy larger than the modulus of its spatial momentum) which is induced by the presence of static monopoles⁴. By integrating over the spectral intensity these two effects do not cancel completely, and one obtains the following integrated black-body anomaly:

$$\delta\rho = \int_0^\infty d\omega I_{\text{SU}(2)} - \int_0^\infty d\omega I_{\text{U}(1)} < 0, \quad (2)$$

where

$$I_{\text{SU}(2)}(\omega) = I_{\text{U}(1)}(\omega) \times \frac{(\omega - \frac{1}{2} \frac{d}{d\omega} G) \sqrt{\omega^2 - G}}{\omega^2} \theta(\omega - \omega^*), \quad (3)$$

and

$$I_{\text{U}(1)}(\omega) = \frac{1}{\pi^2} \frac{\omega^3}{\exp[\frac{\omega}{T}] - 1}. \quad (4)$$

It was argued in [3] that temperature T can be regarded a scalar field which, in a given background cosmology, can be described by the following action:

$$\sqrt{-g} \mathcal{L}_{\text{CMB}} = \left(\frac{\bar{T}_0}{\bar{T}} \right)^3 (k \partial_\mu \delta T \partial^\mu \delta T - \delta\rho(T)), \quad (5)$$

where k is a coefficient in need of empirical determination, and \bar{T} is the mean temperature at a given redshift z . Defining a function $\hat{\rho}(T, T_0)$ as

$$\delta\rho = T_0^2 \hat{\rho}, \quad (6)$$

varying the action associated with Eq. (5) w.r.t. $\delta T = T - \bar{T}$, and linearizing the resulting equation of motion yields:

$$\partial_\mu \partial^\mu \delta T - \frac{3}{\bar{T}} \partial_\tau \bar{T} \partial_\tau \delta T + \frac{\bar{T}_0^2}{k H_0^2} \left[\frac{1}{2} \frac{d^2 \hat{\rho}}{dT^2} \Big|_{T=\bar{T}} \delta T + \frac{1}{2} \frac{d\hat{\rho}}{dT} \Big|_{T=\bar{T}} \right] = 0. \quad (7)$$

⁴W.r.t. the defining SU(2) Yang-Mills fields these monopoles are magnetic. They have a dual interpretation in the Standard Model, and thus are electrically charged w.r.t. U(1)_{em}.

To arrive at Eq. (7), the coordinate transformation

$$\tilde{x}_0 \equiv \tau = H_0 t, \quad \tilde{x}_i = \frac{da}{dt} x_i, \quad (i = 1, 2, 3) \quad (8)$$

was performed. Notice the extremely large factor $(\bar{T}_0/H_0)^2 \sim 10^{60}$ in front of the square brackets in Eq. (7). This factor arises because we chose to measure time τ in units of the age of the Universe, distances from the origin \tilde{x}_i in units of the actual horizon size $H^{-1} = a/\frac{da}{dt}$ (as long as $|\tilde{x}_i|$ is sufficiently smaller than unity), and temperature in units of $T_0 = 2.35 \times 10^{-4}$ eV.

By assuming spherical symmetry for the fluctuation δT , which is relevant for an analysis of the cosmic dipole, Eq. (7) simplifies as:

$$\begin{aligned} 0 = & \partial_\tau \partial_\tau \delta T - \left(\frac{da}{a d\tau} \right)^2 \left[\partial_\sigma \partial_\sigma \delta T + \frac{2}{\sigma} \partial_\sigma \delta T \right] - \frac{3}{\bar{T}} \partial_\tau \bar{T} \partial_\tau \delta T + \\ & \frac{\bar{T}_0^2}{k H_0^2} \left[\frac{1}{2} \frac{d^2 \hat{\rho}}{dT^2} \Big|_{T=\bar{T}} \delta T + \frac{1}{2} \frac{d\hat{\rho}}{dT} \Big|_{T=\bar{T}} \right]. \end{aligned} \quad (9)$$

In Eq. (9) we have introduced $\sigma \equiv \sqrt{\tilde{x}_1^2 + \tilde{x}_2^2 + \tilde{x}_3^2}$. Erroneously, a factor $1/a^2$, $a = \frac{\bar{T}_0}{\bar{T}}$ being the scale factor normalized to unity today, was missing in Eq. (18) of [3] in the prefactor $-H^2/H_0^2 = -\left(\frac{da}{a d\tau}\right)^2$ of the term containing spatial derivatives, and the function $\hat{\rho}$ was used as obtained in the approximation $p^2 = 0$. The main goal of the present article is to investigate to what extent these unaccuracies affect the result δT in the spatially spherically symmetric case.

Eq. (9) is a two-dimensional wave equation with additional terms arising on one hand due to the time-dependence of the cosmological background $(-\frac{3}{\bar{T}} \partial_\tau \bar{T} \partial_\tau \delta T)$ and on the other hand due to the presence of the black-body anomaly: The term $\frac{1}{2} \frac{\bar{T}_0^2}{k H_0^2} \frac{d^2 \hat{\rho}}{dT^2} \Big|_{T=\bar{T}} \delta T$ will be referred to as ‘restoring term’, and the term $\frac{1}{2} \frac{\bar{T}_0^2}{k H_0^2} \frac{d\hat{\rho}}{dT} \Big|_{T=\bar{T}}$ will be referred to as ‘source term’ in the following.

3 Old versus corrected results

The background cosmology used to evolve δT according to Eq. (9) is the same spatially flat Λ CDM model that was used in [3].

In Fig. 1 we plot $\delta \rho$ as a function of redshift $z = a^{-1} - 1$ as obtained in the approximation $p^2 = 0$ [3] (dashed line) and selfconsistently [9] (solid line). In Fig. 2 we plot $\frac{d\delta \rho}{dT} \Big|_{T=\bar{T}}$ as a function of redshift as obtained in the approximation $p^2 = 0$ [3] (dashed line) and selfconsistently [9] (solid line). There is hardly any visually discernable difference between the results for the ‘restoring term’ in Eq. (9) as obtained in the approximation $p^2 = 0$ [3] and selfconsistently [9]. In Fig. 3 a plot of $\frac{\delta T}{\bar{T}}$ is shown as a function of z for fixed distances $\sigma = 0.05; 0.5$, $k = 0.01868 \bar{T}_0^2/H_0^2$, and a width w of the initial Gaussian of $w = 10^{-2}$ at $z_i = 20$. The approximation $p^2 = 0$ [3] together with the erroneous version of Eq. (9) corresponds to the dashed

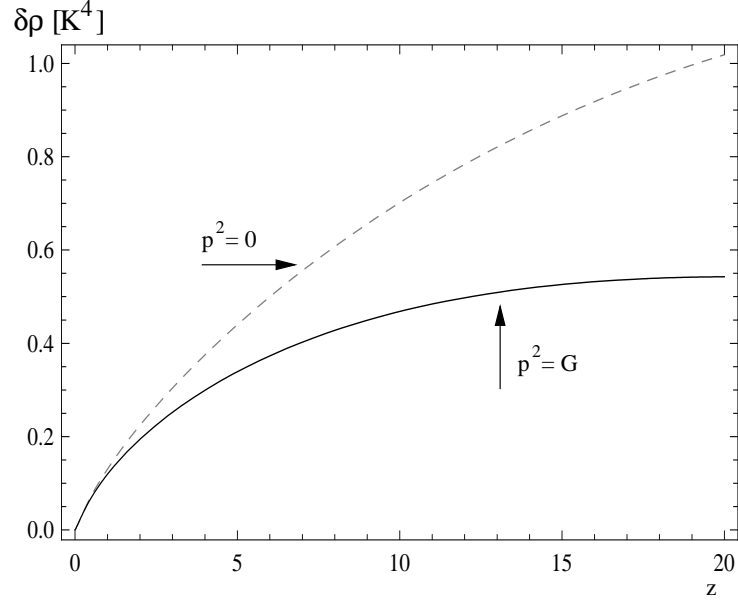


Figure 1: The difference $\delta\rho$ between integrated modified black-body spectral intensity and the integrated Planckian spectrum as a function of redshift z . The approximation $p^2 = 0$ [3] corresponds to the dashed line while the case of the self-consistently determined mass shell [9] is depicted by the solid line.

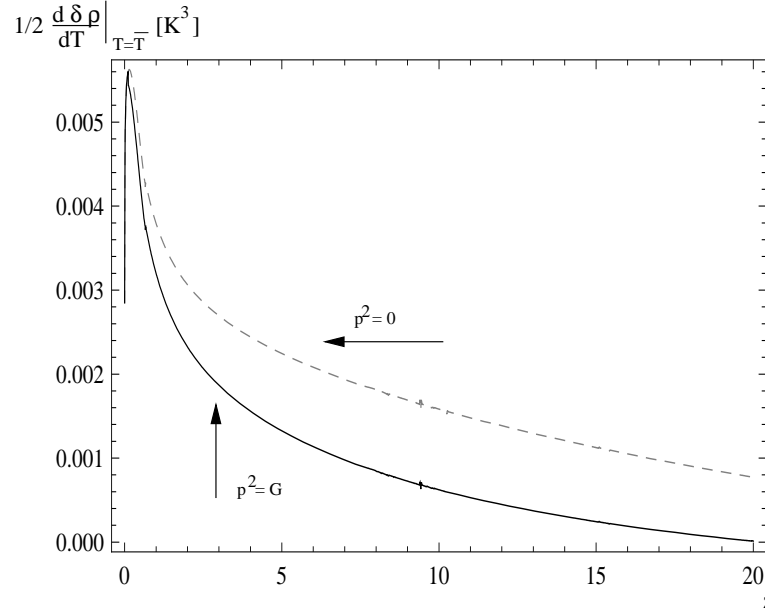


Figure 2: The ‘source term’ $\frac{1}{2} \frac{d\delta\rho}{dT} \Big|_{T=\bar{T}}$ in Eq.(9) as a function of redshift z . The approximation $p^2 = 0$ [3] corresponds to the dashed line while the case of the selfconsistently determined mass shell [9] is depicted by the solid line.

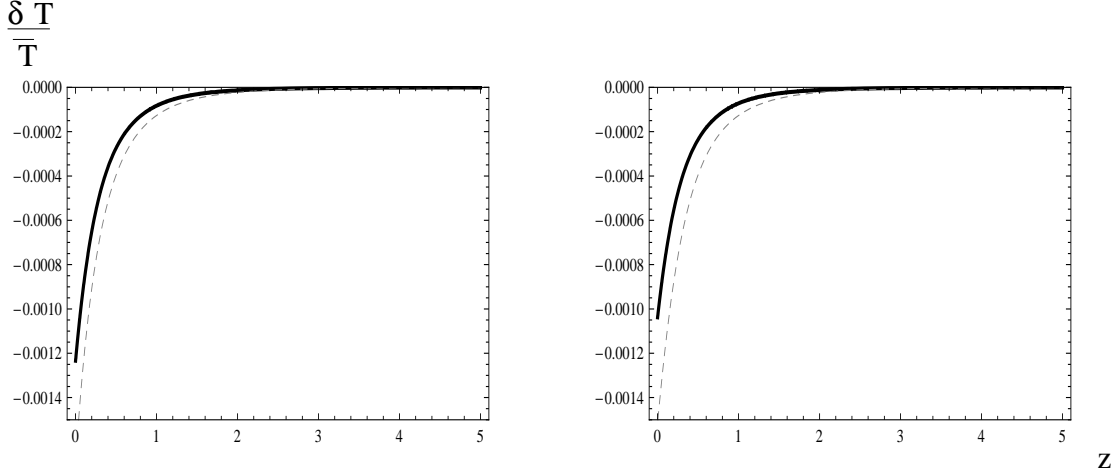


Figure 3: $\frac{\delta T}{\bar{T}}$ for two distances $\sigma = 0.05$ (left panel) and $\sigma = 0.5$ (right panel) as a function of z for $k = 0.01868 \bar{T}_0^2/H_0^2$. The width w of the initial Gaussian is $w = 10^{-2}$ at $z_i = 20$. The approximation $p^2 = 0$ [3] together with the erroneous version of Eq. (9) corresponds to the dashed line while the case of the selfconsistently determined mass shell [9] together with the proper evolution equation Eq. (9) is depicted by the solid line.

line while the case of the selfconsistently determined photon mass shell [9] together with the proper evolution equation Eq. (9) is depicted by the solid line. In Fig. 4 plots of the profiles $\frac{\delta T}{\bar{T}}$ at $z = 1$ and $z = 0$ are shown when computed both with the erroneous version of Eq. (9) together with the approximation $p^2 = 0$ and with the proper equation (9) and the case of the selfconsistently determined photon mass shell. In Fig. 5 plots of the profiles $\frac{\delta T}{\bar{T}}$ at $z = 0$ are shown when computed both with the erroneous version of Eq. (9) together with the approximation $p^2 = 0$ and with the proper equation (9) and the case of the selfconsistently determined mass shell. Notice that $k = 0.01868 \bar{T}_0^2/H_0^2$ in the former case while the k -value for the latter case was adjusted such that the two profiles coincide at $\sigma = 0$. Considering the empirical uncertainty of k [3] this adjustment is admissible. Obviously, the difference between the two curves is small.

4 Summary and conclusions

In this work we have investigated to what extent a calculational error in the derivation of the evolution equation for temperature fluctuations δT and an improvement [9] in determining the black-body-anomaly related terms in this equations affects the results for the evolution of temperature profiles in the spatially spherically symmetric case [3]. We have found that only very small quantitative deviations take place which practically can be absorbed in the empirical uncertainty of the normalization of the kinetic versus the potential term in the associated action (5).

To make contact with the observed large-angle anomalies in the CMB it would be interesting to compute the dipole-subtracted angular correlation function $C(\theta) =$

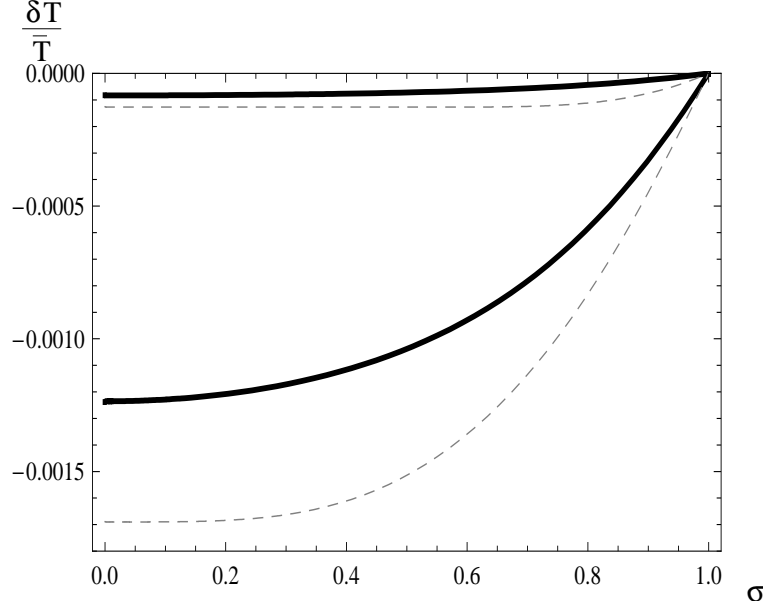


Figure 4: The profile $\frac{\delta T}{\bar{T}}$ at $z = 1$ (upper curves) and $z = 0$ (lower curves). The solid lines correspond the case of the selfconsistently determined mass shell [9] together with the proper evolution equation (9) while the dashed lines are for the approximation $p^2 = 0$ [3] together with the erroneous version of Eq. (9). Again, we have used $k = 0.01868 \bar{T}_0^2/H_0^2$.

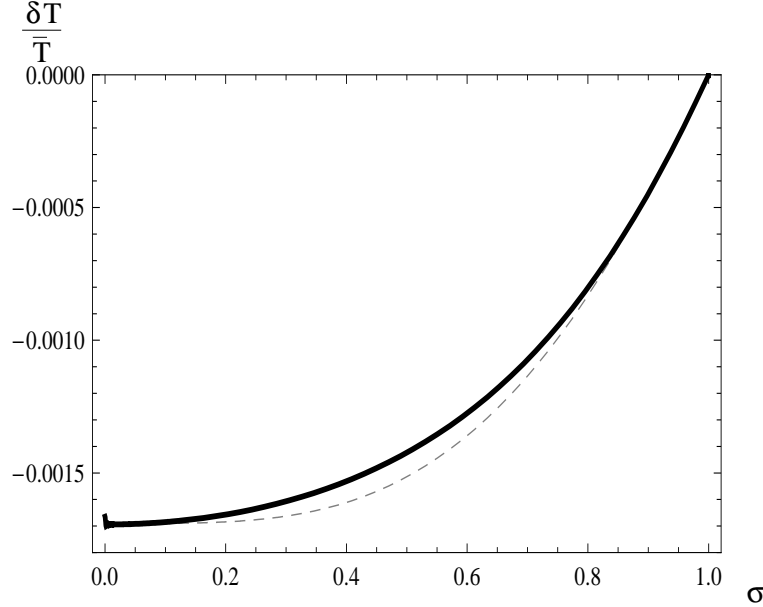


Figure 5: The profile $\frac{\delta T}{\bar{T}}$ at $z = 0$. The dashed line is for the approximation $p^2 = 0$ [3] together with the erroneous version of Eq. (9) and $k = 0.01868 \bar{T}_0^2/H_0^2$. The solid line is obtained for the case of the selfconsistently determined mass shell [9] together with the proper evolution equation (9) and $k = 0.0136 \bar{T}_0^2/H_0^2$. This k -value is generated by demanding that the function $\frac{\delta T}{\bar{T}}$ coincides with the dashed line at $\sigma = 0$.

$\langle \delta T \delta T \rangle$. In the approach discussed in [3] and again applied here this would be done by relaxing the assumption of spherical symmetry made in Eq. (9) to compute an ensemble of present CMB maps of temperature fluctuations (solutions to Eq. (7) minus solutions to Eq. (9), labelled by the initial conditions and by subsequently averaging over this ensemble and over pairs of integrals over lines of sight separated by the angle θ . Due to the rapid build-up of the spherical profile at $z \sim 1$ we expect an anomalous suppression of large-angle correlations and a statistically significant alignment of the low multipoles.

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